

# Throughput Analysis of WLANs in Saturation and Non-Saturation Heterogeneous Conditions with Airtime Concept

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**SUMMARY** This paper presents an analytical model for network throughput of WLANs, taking into account heterogeneous conditions, namely network nodes transmit different length frames with various offered load individually. The airtime concept, which is often used in multi-hop network analyses, is firstly applied for WLAN analysis. The proposed analytical model can cover the situation that there are saturation and non-saturation nodes in the same network simultaneously, which is the first success in the WLAN analyses. This paper shows the network throughput characteristics of four scenarios. Scenario 1 considers the saturation throughputs for the case that one or two length frames are transmitted at the identical offered load. Scenarios 2 and 3 are prepared for investigating the cases that all network nodes transmit different length frames at the identical offered load and identical length frames at the different offered loads, respectively. The heterogeneous conditions for not only frame length but also offered load are investigated in Scenario 4.

**key words:** WLAN, throughput, analysis, frame length, offered load, heterogeneous conditions

## 1. Introduction

Wireless Local Area Network (WLAN) has been widely deployed [1], [2], and applications in WLAN become diversified with the increase in users. It can be stated that WLAN is often in heterogeneous conditions, namely, network nodes transmit different length frames with various offered load individually. Additionally, the situations that there are saturation and non-saturation nodes in the same network simultaneously often appear. It is, however, not easy to comprehend behaviors and performance of the network nodes in the heterogeneous conditions. Analytical expressions are useful and helpful for predicting the network performance.

There are many analytical models of IEEE 802.11 WLAN [3]–[7]. The Markov-chain model presented by Bianchi [3] is a pioneer and well-known model for expressing WLAN throughput. Bianchi's model expresses the backoff-timer state in the IEEE 802.11 Distributed Coordi-

nation Function (DCF). Since the proposal of the Bianchi's model, many extended versions of Bianchi's model have been presented, which considered coupling effect [4], capture effect [5], and non-saturated condition [6]. These analytical models are based on the assumption that all the network nodes work for the homogeneous conditions, namely transmit identical length frame with the same frame occurrence probability. Recently, a throughput model under the heterogeneous conditions was presented in [7]. This model is also the extended version of the Bianchi's model and it is possible to obtain network performance by expressing the network-node operations individually. The analytical model in [7], however, still has a limitation of network situations. For example, it is impossible to express the network performances when there are saturation and non-saturation nodes in the same network simultaneously. This is because the model in [7] is based on the assumption that all the network nodes are non-saturated.

On the other hand, multi-hop network analyses have been carried out actively. In most WLAN analyses, the major factor of the throughput degradation is frame collisions due to concurrent frame transmissions. In the multi-hop networks, however, the major factor of the throughput degradation is not concurrent-transmission frame collisions but frame collisions due to hidden nodes. Additionally, the carrier-sensing relationships among network nodes also limit the network throughput. For expressing the frame collisions due to hidden nodes and the carrier-sensing relationships, the time shares of transmission, carrier sensing, and idle states at each node are useful. The airtime concept appeared from these backgrounds [8]–[11]. The multi-hop network analyses have been progressed based on the airtime concept, which follows the different way from WLAN analyses. By using airtimes, it is possible to express transmission probability, frame-collision probability, carrier-sensing duration, and frame-existence probability with respect to each network node in simple forms [11]. None, however, has considered that the airtime concept is applied to WLAN analysis. This is because the airtime was proposed for expressing the hidden node collisions and carrier-sensing relationships.

The end-to-end delay analysis of string-topology multi-hop networks was carried out in [11]. It is noticed that the analysis in [11] can express the network performance even when the saturation and non-saturation nodes coexist in the same flow. The results in [11] suggest that the airtime con-

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cept may be suitable to express heterogeneous conditions in WLANs as well as multi-hop networks, which is our major idea of this paper.

This paper presents a new analytical model for network throughput of WLANs, taking into account heterogeneous conditions. The airtime concept is applied for expressing WLAN analysis. By applying the airtime concept, it is possible to express frame-existence probabilities of network nodes as a function of the airtimes. Additionally, our analytical model can cover the situation that both saturation and non-saturation nodes coexist in the same network simultaneously. For showing the validity of the analytical expressions, this paper discusses the network throughput characteristics of four scenarios with the heterogeneous conditions.

## 2. Related Works

### 2.1 Analysis of IEEE 802.11 WLANs

There are many analytical models of IEEE 802.11 WLANs [3]–[7]. One of the most famous models for WLANs is presented by Bianchi [3], which is a Markov-chain model for expressing the backoff timer decrement in the IEEE 802.11 Distributed Coordination Function. The Bianchi's model and its extended versions assumed that the network is in the homogeneous condition. We can obtain simple expressions in the homogeneous conditions because all network nodes operation can express with the identical forms on the probability representations.

Recently, an analytical model of WLANs for heterogeneous conditions is proposed in [7]. It is possible to derive the analytical expressions in the heterogeneous conditions by considering the network node individually. Obviously, all the nodes in WLANs have identical transmission probability, frame-collision probability in the saturation condition, it is assumed in [7] that network nodes are in the non-saturation conditions. In real networks, the situation that saturation and non-saturation nodes coexist in the network simultaneously may often appears. There is, however, no analytical expression, which is valid for such kind of situation.

### 2.2 Analysis of Multi-Hop Network Based on Airtime Concept

On the other hand, there are also many analytical models of multi-hop networks [8]–[11]. For expressing the frame collisions due to hidden nodes and the carrier-sensing relationships among the nodes, one of the effective approaches of multi-hop network analysis is the use of airtimes [8], which are time-shares of the network-node state. The multi-hop network analyses have been progressed based on the airtime concept, which follow the different way from WLAN analyses. This is because the airtime expressions are very useful and convenient for modeling the networks including many hidden node relationships. The first analysis using airtime assumes that the analysis object is sufficiently long-hop

string topology network with network-saturation condition. By taking into account long-hop network, the edge problem can be ignored and simple analytical forms can be obtained. In short-hop network, edge problem should be considered [9]–[11]. In the multi-hop networks, there are both saturation and non-saturation nodes simultaneously even when the network throughput is saturated [11]. Namely, the airtime concept can be adopted the coexistence states of saturation and non-saturation nodes. This means that airtime concept is also suitable to WLAN analysis for heterogeneous condition, in particular, which is our claim in this paper. There is, however, no analytical consideration with different frame lengths in the multi-hop network analysis because most of multi-hop network analysis focuses one flow string-topology network [8]–[11].

## 3. Throughput Analysis of WLAN with Airtime Concept

The purpose of this paper is to obtain analytical expressions of network throughputs for WLANs for heterogeneous conditions. It is considered that network nodes transmit different length frames with various frame occurrence probabilities. The main idea of our analysis is to apply the airtime concept for throughput analysis of WLANs. In our knowledge, there is no WLAN analysis with airtime concept. It is expected that the airtime expressions may be suitable to the WLAN analysis with heterogeneous conditions. For obtaining network throughputs, transmission probabilities, frame-collision probabilities, frame existence probabilities are expressed with respect to each node.

Figure 1 shows the network topology, which is analyzed in this paper. This paper focuses on WLANs with one access point (AP), and  $N$  nodes. In addition, the analysis in this paper is based on the following assumptions:

1. Network nodes generate UDP frames with constant frame occurrence probabilities, which follow the Poisson distribution.
2. A network node is in the carrier-sensing range of all the other nodes and the AP.
3. The channel condition is ideal. Therefore, transmission failures occur due to only frame-collisions.

It can be considered in this paper that there are saturation and non-saturation nodes simultaneously, which is the difference of the assumptions between [7] and this paper. It is difficult to obtain analytical expressions with this situation by extending the model in [7]. Because the queuing

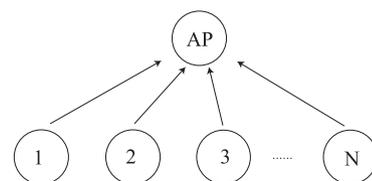


Fig. 1 Example of a network.

model in [7] is not suitable to express both the saturation and non-saturation conditions.

### 3.1 Transmission Airtime

The transmission airtime is the time share for frame transmission, which includes durations of both the successful and failure transmissions [8]–[11]. Concretely, the transmission airtime of Node  $i$  is defined as

$$X_i = \lim_{Time \rightarrow \infty} \frac{S_i}{Time}, \quad (1)$$

$S_i$  is the sum of the durations of the DATA-frame transmissions (*DATA*), ACK frame-transmissions (*ACK*), Distributed InterFrame Space (*DIFS*), and Short InterFrame Space (*SIFS*).

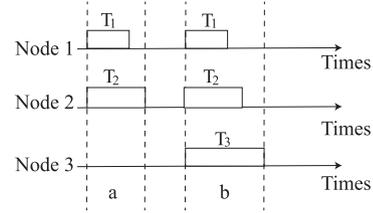
### 3.2 Frame-Collision Probability and Collision Airtime

A frame collision occurs when multiple nodes start to transmit a frame, namely, the back-off timer of the multiple nodes become zero simultaneously. When the transmission probability of Node  $i$  at channel-idle state is expressed as  $\tau_i$ , the frame-collision probability transmitted by Node  $i$  is obtained as

$$\gamma_i = 1 - \prod_{\substack{j=1 \\ j \neq i}}^N (1 - \tau_j). \quad (2)$$

It is necessary to consider the lengths of the collided frames. Figure 2 shows examples of frame collision occurrences in the different frame lengths. When a frame collision occurs, all the nodes cannot go back to the back-off timer countdown state until the longest frame transmissions are ended. The durations for the longest frame transmission in the collided frames are called the frame-collision duration. In Fig. 2, the frame-collision durations are  $T_2$  and  $T_3$  for the frame collisions a and b, respectively.

The probability that a transmitted frame from Node  $i$  collided with  $m$  ( $m < N$ ) nodes is expressed as



**Fig. 2** Example of frame-collisions due to concurrent-frame-transmission.

$$\gamma_{(i,m)} = \sum_{\substack{j_1=1 \\ j_1 \neq i}}^N \sum_{\substack{j_2=1 \\ j_2 \neq i \\ j_2 \neq j_1}}^N \dots \sum_{\substack{j_m=1 \\ j_m \neq i \\ j_m \neq j_1 \\ \dots \\ j_m \neq j_{m-1}}}^N \left[ \tau_{j_1} \tau_{j_2} \dots \tau_{j_m} \prod_{\substack{l=1 \\ l \neq i \\ l \neq j_1 \\ \dots \\ l \neq j_m}}^N (1 - \tau_l) \right]. \quad (3)$$

Obviously, the frame-collision probability of Node  $i$  satisfies

$$\gamma_i = \sum_{m=1}^{N-1} \gamma_{(i,m)}. \quad (4)$$

The ratio of expected value of frame-collision time to one transmission duration of Node  $i$ ,  $C_i$ , can be expressed as (5), where  $T_i = DIFS + DATA_i + SIFS + ACK$ . From the above preparations, the collision airtime of Node  $i$  can be expressed as

$$W_i = \gamma_i X_i C_i. \quad (6)$$

We would like emphasize that  $W_i - X_i \gamma_i$  is included in the carrier-sensing airtime of Node  $i$ , which is explained in 3.3. By using collision airtime, it is possible to express the collision state with different length frames.

### 3.3 Carrier-Sensing Airtime

The carrier-sensing airtime is the time-share of the carrier-sensing state. Basically, the carrier-sense airtime of Node  $i$  is the total transmission airtimes of the carrier-sensing nodes. For obtaining the carrier-sensing airtime of Node  $i$  accurately, however, it is necessary to consider the overlaps of the multiple transmissions due to the frame collisions

$$\begin{aligned} C_i &= \frac{T_{max_i}}{T_i} \\ &= \frac{1}{T_i} \sum_{m=1}^{N-1} \frac{\gamma_{(i,m)}}{\gamma_i} \sum_{\substack{k_1=1 \\ k_1 \neq i}}^N \sum_{\substack{k_2=1 \\ k_2 \neq i, k_2 \neq k_1}}^N \dots \sum_{\substack{k_m=1 \\ k_m \neq i, k_m \neq k_1, \dots, k_m \neq k_{m-1}}}^N \left[ \frac{\tau_{k_1} \tau_{k_2} \dots \tau_{k_m} \prod_{\substack{l=1, l \neq i \\ l \neq k_1, \dots, l \neq k_m}}^N (1 - \tau_l)}{\gamma_{(i,m)}} \max(T_i, T_{k_1}, \dots, T_{k_m}) \right] \\ &= \frac{1}{T_i \gamma_i} \sum_{m=1}^{N-1} \sum_{\substack{k_1=1 \\ k_1 \neq i}}^N \sum_{\substack{k_2=1 \\ k_2 \neq i, k_2 \neq k_1}}^N \dots \sum_{\substack{k_m=1 \\ k_m \neq i, k_m \neq k_1, \dots, k_m \neq k_{m-1}}}^N \left[ \tau_{k_1} \tau_{k_2} \dots \tau_{k_m} \prod_{\substack{l=1, l \neq i \\ l \neq k_1, \dots, l \neq k_m}}^N (1 - \tau_l) \max(T_i, T_{k_1}, \dots, T_{k_m}) \right] \end{aligned} \quad (5)$$

among other nodes. Therefore, the carrier-sense airtime of Node  $i$  can be expressed as

$$Y_i = \sum_{\substack{j=1 \\ j \neq i}}^N \left\{ X_j(1 - \gamma_j) + W_j \left[ 1 - \frac{\tau_i}{\gamma_j} \right] \right\} + W_i - X_i \gamma_i. \quad (7)$$

By using airtime, it is possible to express the carrier-sensing state at different length frames, which is also one of the advantages to apply the airtime concept to the WLAN analysis.

### 3.4 Idle Airtime

The idle airtime of Node  $i$  is the time-share of backoff-time decrement or no frame in the buffer. Namely, idle state is the state except transmission and carrier-sensing states. Therefore, idle airtime can be expressed as

$$Z_i = 1 - X_i - Y_i. \quad (8)$$

### 3.5 Transmission Probability and Frame-Existence Probability

According to [12], the transmission probability when Node  $i$  is in idle state at the saturation condition is expressed as

$$G_i = \frac{R_i}{V_i} = \frac{\sum_{s=0}^K \gamma_i^s}{\sum_{s=0}^K \frac{\gamma_i^s B_s}{2}}, \quad (9)$$

where  $R_i$  and  $V_i$  are the mean numbers of frame transmission attempt and backoff-timer (BT) decrement for one frame-transmission success, respectively. Additionally,  $B_s$  is

$$B_s = \begin{cases} 2^s(CW_{min} + 1) - 1, & 0 \leq s \leq K' - 1 \\ 2^{K'}(CW_{min} + 1) - 1 = CW_{max}, & K' \leq s \leq K \end{cases} \quad (10)$$

where  $CW_{min}$  and  $CW_{max}$  are the minimum and the maximum contention-window values respectively,  $K' = \log_2 \frac{CW_{max} + 1}{CW_{min} + 1}$  and  $K$  is the retransmission limit number. It is known from [9] that at the saturation condition the relationship between the transmission and idle airtimes can be expressed as

$$X_i = \frac{G_i T_i Z_i}{\sigma}. \quad (11)$$

For the definition of the frame transmission probability at non-saturation condition, it is necessary to derive the frame-existence probability  $Q_i$ , which is the probability that Node  $i$  has at least one frame in the channel-idle state. This is because  $G_i$  is defined on the assumption of the saturation condition. Because the BT decrement is carried out only when a node has frames in the channel-idle state, an airtime that Node  $i$  decreases the BT in whole time is

$$Q'_i Z_i = \lambda_i \sigma V_i = \frac{\sigma O_i V_i}{P_i}, \quad (12)$$

where  $Q'_i$  is the frame-existence probability at non-saturation condition,  $\lambda_i$  is the frame-occurrence rate,  $O_i = \lambda_i P_i$  is the offered load, and  $P_i$  is the payload size. Additionally,  $\sigma V_i$  means the average spending time of BT decrement for one frame transmission success. (12) is based on the fact that the frame-occurrence number is the same as frame-transmission number in the non-saturation condition. By using  $\lambda_i$ , we have also another expression of transmission airtime as

$$X_i = \lambda_i R_i T_i. \quad (13)$$

From (9), (12), and (13), we have

$$X_i = \frac{G_i Q'_i T_i Z_i}{\sigma}. \quad (14)$$

By using (12), it is possible to express the frame-existence probability as

$$Q_i = \min(1, Q'_i) = \min\left(1, \frac{\sigma O_i V_i}{P_i Z_i}\right), \quad (15)$$

which is valid for both the saturation and non-saturation states. The min function prevents  $Q_i$  from exceeding 1. Note that the frame-existence probability, which expresses both saturation and non-saturation conditions, can be expressed by using airtime. This is the most important reason why the airtime concept is applied to this analysis.

Finally, the general expression of the frame transmission probability at channel-idle state can be obtained from (11), (14), and (15) as

$$\tau_i = G_i Q_i = \frac{\sigma X_i}{T_i Z_i}. \quad (16)$$

### 3.6 Network Throughput

By using the transmission airtime, throughput of Node  $i$  is obtained as

$$E_i = X_i(1 - \gamma_i) \frac{P_i}{T_i} = \tau \sigma (1 - \gamma_i) P_i Z_i. \quad (17)$$

In WLAN, the network throughput is the sum of all the network-node throughputs, which is expressed by

$$E_{total} = \sum_{i=1}^N E_i. \quad (18)$$

From (2), (6)–(8), (14)–(16), we have  $7N$  relationships expressed by the algebraic equations. These equations contain  $7N$  unknown parameters, which are  $\gamma_i$ ,  $W_i$ ,  $Y_i$ ,  $Z_i$ ,  $Q_i$ ,  $X_i$ , and  $\tau_i$ , for  $i = 1, 2, \dots, N$ . It is possible to fix the  $7N$  unknown parameters when the offered loads of all network nodes are given. In this paper, Newton's method is used for solving the algebraic equations [13]. When we can fix the unknown parameters, it is possible to predict network performance, such as throughput, frame-existence probability

**Table 1** System parameters.

MAC header	24 bytes
PHY header	16 bytes
ACK size	10 bytes
Data rate	54 Mbps
ACK bit rate	24 Mbps
SIFS time ( <i>SIFS</i> )	16 $\mu$ sec
DIFS time ( <i>DIFS</i> )	34 $\mu$ sec
slot time( $\sigma$ )	9 $\mu$ sec
$CW_{min}$	15
$CW_{max}$	1023
Retransmission limit ( <i>K</i> )	7

**Table 2** Introduction for four scenarios.

Scenario	Types of frame lengths	Offered loads
1	One or two	Identical
2	Eight	Identical
3	One	Non-identical
4	Eight	Non-identical

and frame-collision probability.

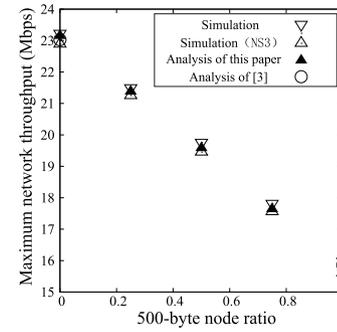
#### 4. Simulation Verification

In this section, the validities of the proposed analytical expressions are evaluated and discussed from comparisons with the simulation results. Table 1 gives system parameters for simulations and analytical predictions, which basically follows the IEEE 802.11a standard [14]. The WLAN with eight nodes is considered with four scenarios which are introduced briefly as given in Table 2. An original simulator, which was implemented by authors, was used in this paper. The original simulator describes only the MAC layer operations. When frame transmissions are overlapped, both the frames are regarded as collided frames. The operations of all the nodes and the access point are expressed in slot time variations. The static routing tables are given for all the network nodes to the access point. Transmission frames yield in each network node with Poisson distribution. The offered load can be calculated from the occurrence frame number. Additionally, throughputs are obtained from the reception frame number at the access point. The source file of our simulator is available on [15]. It was confirmed that the original simulator gave the same WLAN throughputs as NS3 simulator [16]. This paper shows the throughput characteristics from NS3 simulation as well as the original simulator.

##### 4.1 Comparisons with Previous Model Form [3]

In Scenario 1, there are two length frames in the network, namely 500 bytes and 1000 bytes. Figure 3 shows the maximum network throughputs as a function of ratio of 500-byte node number  $H$ .

It is seen from Fig.3 that the maximum network throughput decreases as the short-frame transmission node increases. This is because the data payload decreases as DATA frame length becomes short. In the cases of  $H = 0$  and 1, which mean all the nodes transmit the same-length

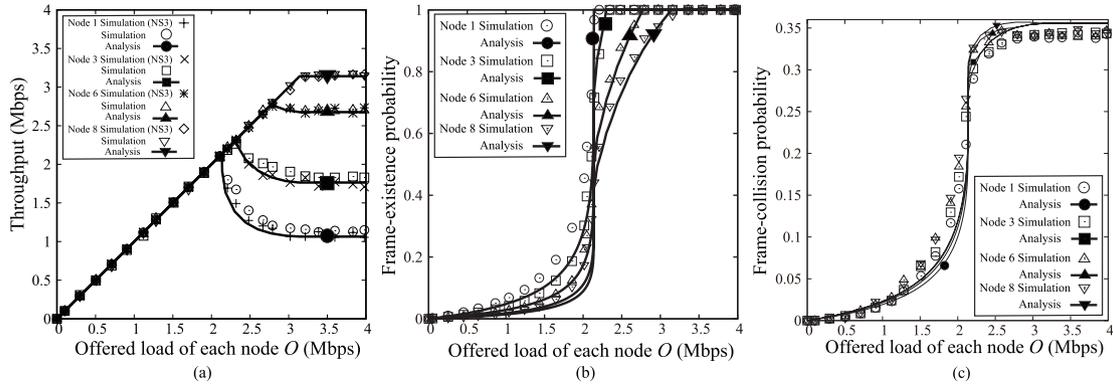
**Fig. 3** Maximum network throughput as a function of ratio of 500-byte node number.

frame, the proposed analytical results agree with the results in the previous papers of [3]. Additionally, the analytical predictions agree with the simulation results in the range of  $0 < H < 1$ . These results show the validity of the proposed analytical model.

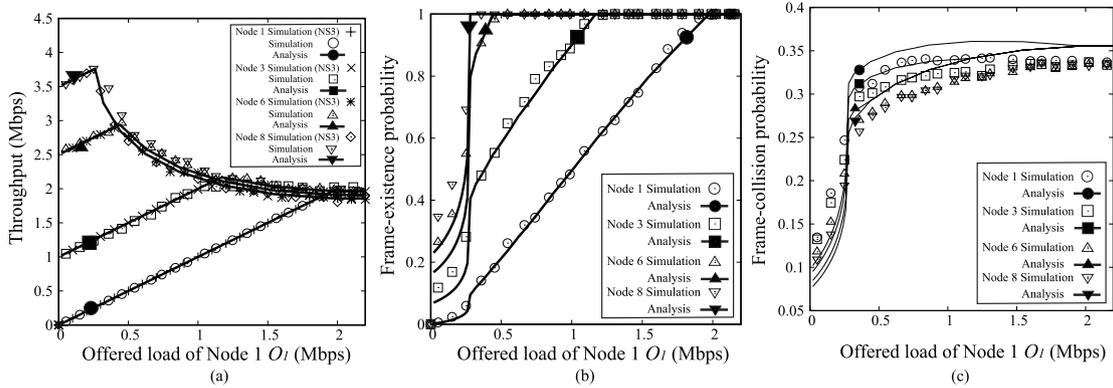
##### 4.2 For Different Frame Lengths and Identical Offered Load

Next, we consider the case that Node  $i$  transmits  $(200 + 100i)$  bytes frames with identical offered loads  $O_i = O$  as Scenario 2. Namely, the frame-occurrence probabilities depend on the nodes. Figure 4 shows throughputs, frame-existence probabilities, and frame-collision probabilities as a function of the offered load for fixed node. It can be seen from Fig 4(a) that the throughputs increase in proportion to the offered load in the range of  $0 < O < 2.15$  Mbps. This is because all the nodes are in non-saturation state, which are confirmed from Fig. 4(b). Namely, it is seen that frame-existence probabilities of all the nodes are less than one in the range. The frame-occurrence probability increases as the frame length decreases because of the identical offered load condition. As a result, the frame-existence probability reaches one firstly at  $O = 2.15$  Mbps. For  $O > 2.15$  Mbps, the frame-existence probabilities of nodes except Node 1 still increase. Therefore, both frame-collision probability and carrier-sensing airtime increase, which is confirmed from Fig. 4(c). As a result, the throughput of Node 1 decreases as offered load increases in the saturation condition. It is also seen from Fig. 4(b) that all nodes are saturated at  $O = 3.15$  Mbps. Therefore, the network throughput and frame-collision probability with respect to each nodes keep the constant value in the range of  $O > 3.15$  Mbps. Additionally, it can be confirmed from Figs. 4(a) and (b) that each node obtains its maximum throughput when the existence probability of the node reaches one.

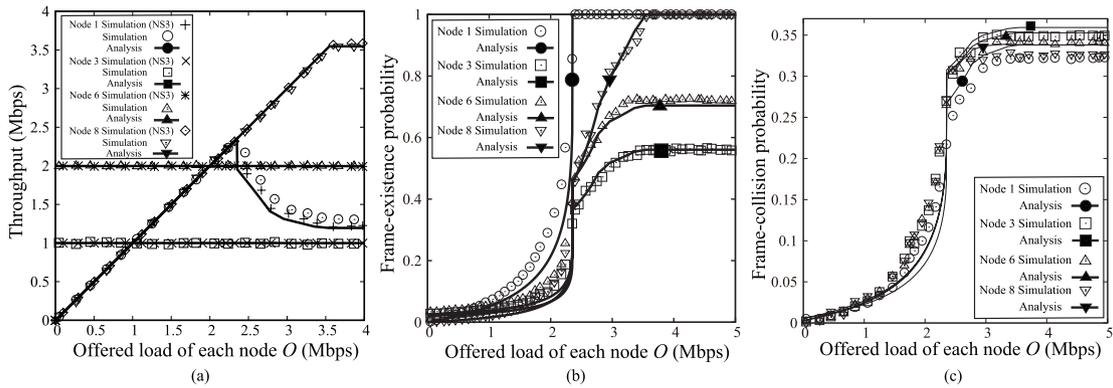
In this scenario, there are saturation and non-saturation nodes in the same network simultaneously in the range of  $2.15 < O < 3.15$  Mbps, which can be seen from Fig. 4(b). It can be stated that the proposed analytical model is valid for the WLAN networks with heterogeneous conditions.



**Fig. 4** Network characteristics as a function of offered load of each node for Scenario 2. (a) Throughput. (b) Frame-existence probability. (c) Frame-collision probability.



**Fig. 5** Network characteristics as a function of offered load of Node 1 for Scenario 3. (a) Throughput. (b) Frame-existence probability. (c) Frame-collision probability.



**Fig. 6** Network characteristics as a function of offered load of each node for Scenario 4. (a) Throughput. (b) Frame-existence probability. (c) Frame-collision probability.

4.3 For Same Frame Length and Non-Identical Offered Load

It is considered that all the nodes transmit 500 bytes frames with non-identical offered loads. The offered load of Node  $i$  is given by  $O_i = (O_1 + (i - 1)/2)$  Mbps. Figure 5 shows throughputs, frame-existence probabilities and frame-collision probabilities as a function of the offered

load  $O_1$  for fixed node. All the nodes are in non-saturation state in the range of  $0 < O_1 < 0.28$  Mbps. From (15), the frame-occurrence probability increases as the offered load increases because of the same frame length condition. It is seen from Fig. 5(b) that Node 8 becomes in the saturation state firstly at  $O_1 = 0.28$  Mbps with the increase in the offered load. In the region of  $O_1 > 0.28$ , the frame-collision probabilities of network nodes are almost constant. All the nodes are in saturation state at  $O_1 = 1.98$  Mbps.

#### 4.4 Co-Existence of Saturation and Non-Saturation Nodes

Next, another scenario is considered. The frame lengths in Scenario 4 are the same as those of Scenario 2. In this scenario, Nodes 3 and 6 transmit frames with a constant offered load, namely  $O_3 = 1$  Mbps and  $O_6 = 2$  Mbps, which are lower than the saturation throughput in Fig. 4. The other nodes transmit frames with identical offered load, namely,  $O_1 = O_2 = O_4 = O_5 = O_7 = O_8 = O$ .

Figure 6 shows the throughputs, frame-existence probabilities, and frame-collision probabilities as functions of  $O$  for the fixed node. It is seen from (15) that the frame-existence probability is affected by the frame-occurrence rate and idle airtime. Similar to Scenarios 2 and 3, the throughput and frame-existence probability of the nodes, whose offered loads are not constant, increase with the increase in the offered load in the non-saturation region. The frame-existence probabilities of non-constant offered load nodes reach one as offered load increase. The frame-existence probabilities of nodes 3 and 6, however, never reach one. It is seen from Fig. 6(a) that network throughput is saturated for  $O > 3.55$  Mbps. It can be stated from these results that there are saturation and non-saturation nodes simultaneously at the saturation network throughput condition.

It is seen from all the results in Figs. 3–6 that the analytical predictions agree with the simulation results quantitatively, which show the validities of the analytical expressions in this paper. Additionally, throughputs of all the scenarios show the complete agreements between the original simulator and NS3, which denote the credibility of the original simulator.

## 5. Conclusion

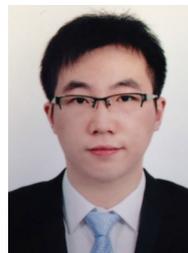
This paper has presented an analytical model for network throughput of WLANs, taking into account frame length and offered load. The airtime concept is firstly applied for WLAN analysis in this paper. Network throughputs can be obtained analytically by expressing frame-collision probabilities and frame-existence probabilities with respect to each node. The proposed analytical model could cover heterogeneous operations of network nodes. Additionally, it is possible to express the network throughput even when buffer-full and buffer-empty nodes coexist. The validities of the analytical expressions are confirmed from quantitative agreement between analytical predictions and simulation results.

## References

- [1] A. Kajackas and L. Pavilanskas, "Analysis of the technological expenditures of common WLAN models," *Electronics and Electrical Engineering*, Kaunas: Technologija, vol.72, no.8, pp.19–24, April 2006.
- [2] Y. Xiao, "Performance analysis of priority schemes for IEEE 802.11 and IEEE 802.11e wireless LANs," *IEEE Trans. Wireless Commun.*,

vol.4, no.4, pp.1506–1515, July 2005.

- [3] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," *IEEE J. Sel. Areas. Commun.*, vol.18, no.3, pp.535–547, March 2000.
- [4] T. Kim and J.-T. Lim, "Throughput analysis considering coupling effect in IEEE 802.11 networks with hidden stations," *IEEE Commun. Lett.*, vol.13, no.3, pp.175–177, March 2009.
- [5] S. Abukharis, R. MacKenzie, and T. O'Farrell, "Non-saturated throughput analysis for a QoS differentiated p-persistent CSMA protocol with the capture effect," *Proc. 2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*, pp.1216–1220, 2015.
- [6] K. Duffy, D. Malone, and D.J. Leith, "Modeling the 802.11 distributed coordination function in non-saturated conditions," *IEEE Commun. Lett.*, vol.9, no.8, pp.715–717, June 2005.
- [7] D. Malone, K. Duffy, and D. Leith, "Modeling the 802.11 distributed coordination function in nonsaturated heterogeneous conditions," *IEEE/ACM Trans. Netw.*, vol.15, no.1, pp.159–172, Feb. 2007.
- [8] P.C. Ng and S.C. Liew, "Throughput analysis of IEEE802.11 multi-hop ad hoc Networks," *IEEE/ACM Trans. Netw.*, vol.15, no.2, pp.309–322, April 2007.
- [9] Y. Gao, D.-M. Chiu, and J.C.S. Lui, "Determining the end-to-end throughput capacity in multi-hop networks," *Proc. SIGMETRICS'06/Performance'06*, pp.39–50, 2006.
- [10] M. Inaba, Y. Tsuchiya, H. Sekiya, S. Sakata, and K. Yagyu, "Analysis and experiments of maximum throughput in wireless multi-hop networks for VoIP application," *IEICE Trans. Commun.*, vol.E92-B, no.11, pp.3422–3431, Nov. 2009.
- [11] K. Sanada, J. Shi, N. Komuro, and H. Sekiya, "End-to-end delay analysis for IEEE 802.11 string-topology multi-hop networks," *IEICE Trans. Commun.*, vol.E98-B, no.7, pp.1284–1293, July 2015.
- [12] A. Kumar, E. Altman, D. Miorandi, and M. Goyal, "New insights from a fixed-point analysis of single cell IEEE 802.11 WLANs," *IEEE/ACM Trans. Netw.*, vol.15, no.3, pp.588–601, June 2007.
- [13] C.T. Kelley, *Iterative Methods for Linear and Nonlinear Equations*, North Carolina State University, Raleigh, NC, 1995.
- [14] "Wireless LAN medium access control (MAC) and physical layer (PHY) specifications," *IEEE Std. 802.11*, Feb. 2012.
- [15] The original simulator, <http://www.s-lab.nd.chiba-u.jp/link/simulator.htm>, accessed May 24, 2016.
- [16] The network simulator-ns3, <http://www.nsnam.org>, accessed May 20, 2016.



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